

## Determination of Hadronic Nuclei Radius in 3DL Model

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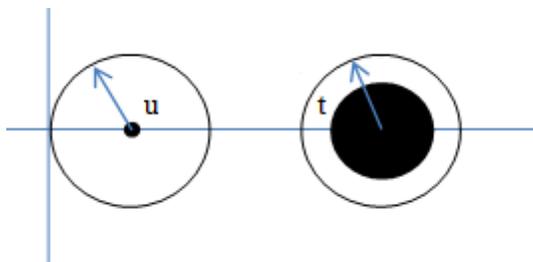
**ABSTRACT**— hadronic nuclei like systems that consist of hadrons that are strongly interacting particles bound states. The kinds of hadronic nuclei that have been made and the years in which they were first identified include proton, neutron, di-baryon, quadroquark, pentaquark, etc. The lifetime of these hadronic nuclei are of the order of 10–20 s, but this is long enough to identify them and study their characteristics by means of their strong interactions. They are available to study only in the beams of particles hyper accelerators. We describe the theory of hadronic nuclei in quantum chromodynamics and quantum electrodynamics based on strong interactions framework. We first provide an introduction to the overlap clustering model (OCM) based on three-dimensional layout (3DL), and then calculate colored charge radius of several hadronic nuclei like proton, neutron, lambda, sigma and omega, etc. We investigate the dynamics of the hadronic nuclei in which hadrons are defined as atomic nuclei in 3DL OCM that presented by the author to describe new definition of atomic nuclei's structures. The general properties of bounding state and the conditions for it are studied via the 3DL OCM.

**KEYWORDS:** Strong interactions, hadronic nuclei, bound state, ground state, gluon, quark.

**ABBREVIATION:** Overlap clustering model; Three-dimensional layout.

### Introduction

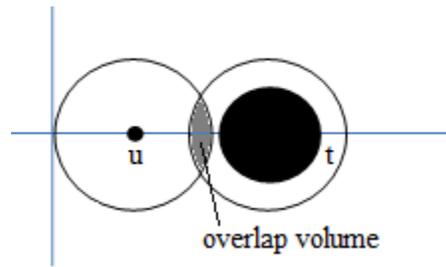
The International theoretical and experimental studies on quarks bound states started at CERN in 2002 and in 2003 we heard about pentaquark [1,2] hadronic structure, charmonium state, heavy mesons [3-6], and exotic baryons that were the most striking event by a photon beam of energy up to 2.4 GeV that very soon confirmed by various groups, including ITEP (DIANA), JLAB (CLAS) and ELSA (SAPHIR). These achievements gave us an opportunity to describe thirteen-year-long research that has seen a cornucopia of new data pouring out and new theory challenges identified. The modern experiments on hadronic structures already have had, and will continue to have, an important role in shaping hadronic physics. Some parts of bound states characteristics [7] and creation of new hadronic structures in hadronic physics based on spherical symmetric color freedom (Fig.1) can be described characteristics of hadronic nucleon in 3DL OCM [8]. This spherical colored charge has the same radius for all quarks that started from the quark mass center. For example: we show the colored charge radius for two different types of quarks with masses: up quark (2.3MeV) and top quark (4.18GeV) (Fig.1). Strong interaction's particles are colored charge (quarks and gluons) [8-11]. Just as electrically charged particles interact by exchanging photons in electromagnetic interactions, colored charge particles exchange gluons in strong interactions. When two quarks are close to one another, they exchange gluons and create a very strong colored force field that binds the quarks together. The force field gets stronger as the quarks get further apart. Quarks constantly change their colored charges as they exchange gluons with other quarks.



**Fig1:** Colored charge radius of up and top quarks

In strong interaction space exist three colored and three corresponding anti-colored charges values. Each quark or gluon has one of three colored and three anti-colored charge values. In this case, there are multi overlapping colored charge volume that created by strong interactions based on gluon- quark interacting effects, which connect quarks to each other. We supposed that this volume is the spherical-symmetric space around each quark and known as colored charge character of hadronic particles. Just as a mix of red, green, and blue light yields white light, in a baryon combination of "red", "green", and "blue" colored charges is color neutral, and in an antibaryon "antired", "antigreen", and "antiblue" is also color neutral. Mesons have neutral color because they carry combinations such as "red" and "antired". In 3DL OCM representation all quarks have the same colored charge radius.

The strong interactions and interchanging gluons between quarks will be appeared in overlapping volume of colored charge near the mass of each quark. In the mesons this volume shows in Fig2:



**Fig2:** strong interaction and colored charge overlapping volume.

The overlapping model in three dimensional system is the interesting theoretically method that can be very useful in definition of hadronic nuclei's structures. In this approach all kinds of quarks have the same characteristics as usual characteristics in other theories (like radius, volume, angular momentum, spin, electric charge, colored charge, and strong force) but, they are different in the particle's mass value so the overlapping volume is the same in all types of quarks. The quarks are interacting with each other by exchanging gluon that origin from the strong interaction principles which are much more complicated than other fundamental interactions. As we know the quarks are held together by their color and confinement characteristics. In 3DL OCM all quarks are alike, whereas there are six flavours and three colors. Notice that there are too many types of hadronic nuclei that include excited states too, but we could make 3DL quarks arrangements just include ground state structure. 3DL model analyzes the interaction between different configurations of quarks in the hadronic nuclei with new definition of layer's arrangement that based on the first configuration of meson-two quarks (two core) and proton/neutron- three quarks (three core) structures. This configuration depends on the total number of quarks that we want to create in the bound states. This arrangement should be such that the initial hadronic two and three core structures of basic hadronic's nuclei figured on the spherical-symmetric shape in needed layers[12]. We describe the main method to find the best arrangement of hadronic nuclei based on 3DL OCM.

## 2. Colored overlapping space in hadrons

The structure of hadronic systems has been depended on the total number of quarks that bounded together. The structural properties of hadronic nuclei with 3DL model are known to describe construction of new hadronic bounding systems. In this method at the first we should describe overlapping states of summarizing quarks arrangement that includes in different interactions layers. In this representation we give that hadronic system should have spherical-symmetric shape, where strongly bounded by strong interaction's characteristics and quarks arrange their place as a particle around the initial quarkonic core in more than three quarks bound state like quadroquark and pentaquark. Such overlaps arrangement may be a general feature of 3DL OCM caused by the orientation of the hadronic nucleons (quarks) with overlaps and distributing them among the initial quarkonic core. Therefore, in order to get spherical-symmetric shape, hadronic nuclei should maximize the overlaps amount values. We assume that overlaps in all connections are the same. Thus, the ground states of three quarks may resemble embedded in the three-dimensional coordinate that strongly bound together because of spherical-symmetric exact shape and based on gluons interactions. Therefore, as we know quarkonium is the simple tightly-stable hadronic bound states. The stability of the quarkonium is determined by binding energy, i.e. the amount of binding energy that it needs per quark. This method and its possibility to present overlap model for light and heavy hadronic systems can describe creation of ground states. So the 3DL model has ability for finding the best arrangement. It is easier finding the best arrangement with spherical-symmetric structure, certain possess towards overlaps model and characteristics for hadronic nuclei. So the overlap 3DL process by overlaps in three-dimensional structures should be sensitive to the spherical-symmetric shape around the initial core, i.e., it connects quarks with the maximum numbers of overlaps in surface layers (x, y) and in the vertical layer (z). In this case the 3D simulation software's and programs have been used to design and arrange hadronic structures towards 3DL overlap model. 3DL overlap approach revealed that quarks how can connect to each other and how they find the best structure for completing spherical-symmetric characteristics. So our started point is the simple hadronic nuclide, i.e., two quarks and three quarks system, which are well known as mesons and baryons. We try to present 3DL model for these structures. After this section, we will present the basic and initial quarkonik core, i.e., mesons and baryons. The 3DL model can be useful in describing available quarks bounding systems. Now we describe 3DL approach based on shell and liquid drop model which accounts for many feature of the binding energy. Using principles of this model, quarks are added continuously to make hadronic nuclei via overlap's characteristics that they start to bind from lowest-energy layers in 3DL model. As, we described in previous paragraphs, to make hadronic structures in 3DL model quarks should be formed spherical-symmetric shape. In 3DL approach we neglected from spin projection, which means all quarks have the same spin magnitude. This assumption permitted us to keep Pauli principles and it is not violated in 3DL model.

In this model, nucleons will monotonically fill layers from the initial hadronic core in all dimensions for stability and instability hadrons. This concept is similar to that found in an atom orbits where filled with set of electron, so, quarks with attention on the spherical-symmetric characteristics will fill layers in the 3DL structure.

### 3. Bound state mass spectrum

Theoretical models of the 3DL approach encounter two important principles: 1) colored charge radius of quarks are similar to each other, 2) structure of hadronic nuclei based on gluon, the boson of strong interactions and photon the boson of electromagnetic interactions, but in the first step, we just decided to take electromagnetic interactions in overlap modeling, which is defined by Coulomb's Law. As we know electromagnetic interactions independent on the mass of interacted particles. In order to describe color interaction and creation of hadronic nuclei system and then define and predict the colored charge radius by using 3DL model, we should build the initial hadronic nucleonic core, i.e., mesonic core and baryonic core bounding states. In this case, we introduce the main figure in 3DL model that describe mesonic core with two quarks. The pions ( $\pi^+(u\bar{d}), \pi^-(\bar{u}d)$ ) are the lightest most stable meson, and proton ( $uud$ ) is the lightest and most stable baryon. Thus in the 3DL model pion is the basic mesonic core. We start our configuration method from the meson and baryonic cores in the ground state [1]. Overlapping volume in the mesonic core formed because of strong interactions principles, i.e., quark-gluon exchange in bounding states. Based on 3DL model, the colored charge volume around quarks should have the overlapping form like figure 1:

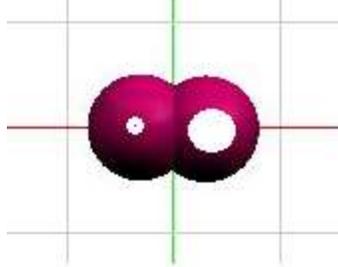


Fig. 3. The core structure of mesons in 3DL model that has one overlap volume in xy-surface.

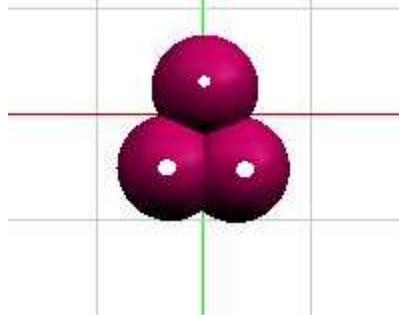


Fig. 4. The core structure of baryons in 3DL model that has three overlaps volumes in xy-surface.

Meson's and baryon's mass have relation with quarks mass and colored charge radius, so they should use to determine the radius of hadronic nuclei in 3DL model that determine by these equations for mesons and baryons based on initial cores one can carry out by using:

$$r_{meson/baryon} = m_{meson/baryon} \times \frac{m_{(\pi/p)}}{r_{(\pi/p)}} \times \frac{r_{q_{heavy}}}{rq_d} \times \frac{\kappa}{\gamma} \quad (1)$$

here  $r_{q_{heavy}}$  is the heaviest quark inside meson and  $\kappa$  is the number of overlaps,  $\gamma$  is the baryon's symmetric coefficient mass (mass correction's coefficient). We will have three corrections in equation (1) if baryons contain massive quarks, i.e. ( $c$  and  $b$ ) 1:  $r_{qd}$  is replaced with  $q_s$ , 2: if heavy baryon has two heavy similar or no similar quarks ( $c, b, s$ ) then  $\gamma = 2$  and if heavy baryon has two light quarks ( $u, d$ )  $\gamma=0.66$ , otherwise for baryons and mesons  $\gamma=1$ . The theoretical description of the radius value of mesons and baryons can be carried out, mainly in the framework of strong interactions and perturbation theory of quantum chromodynamics. The response of the quark mass, quark radius, hadronic core mass, and the number of colored charge overlapping values can be determined and predicated the radius of hadrons in the ground states that directly depend on the bounding mass of systems. For determination of bound state mass we should solve Schrödinger equation by quarks potential model. Based on principles of

strong interaction, the potential model of colored quarks for the masses, decay rates, and other properties of mesons and baryons can be empirically represented as follows:

$$V(r) = -\frac{\alpha_s}{r} + \sigma r \tag{2}$$

$\alpha_s, \sigma$  are the electromagnetic and strong. Therefore, radius of hadrons should determine by mass of hadronic nuclei, radius of quarks on the basis of mesonic and baryonic cores. In this study, we define the bounding mass equation of hadrons in ground state based on quantum chromodynamics and quantum field theory [11]. We carried out the bounding mass of hadronic systems on the asymptotical Behaviour of scalar particles in external gauge field. In this case, at first step we use loop function of two scalar particles with different masses  $m_1$  and  $m_2$  with average on external statistical field is considered and the Green's function is used. The polarization operator in an external electromagnetic field looks like [13-18]:

$$\Pi(x-y) = G_{m_1}(x, y|A)G_{m_2}^*(y, x|A)_A \tag{3}$$

The Green's function  $G(x, y|A)$  for scalar particles in an external field is determined from the equation

$$\left[ \left( i \frac{\partial}{\partial x_a} + \frac{g}{c\hbar} A_a(x) \right)^2 + \frac{c^2 m^2}{\hbar^2} \right] G(x, y|A) = \delta(x-y) \tag{4}$$

where  $m$  is the mass of a scalar particle, and  $g$  is the coupling constant of interaction. After averaging over the field  $A_a(x)$  and using functional integral and also the Feynman Path integral trajectories in nonrelativistic quantum mechanics [19-21] for the motion of two particles with masses  $\mu_1$  and  $\mu_2$ . The interaction of these particles is described by the potential and non-potential interaction. So, the mass of the bound state is determined by the equation [18-24]:

$$M_{meson} = \sqrt{m_1^2 - 2\mu^2 \frac{\partial E(\mu)}{\partial \mu}} + \sqrt{m_2^2 - 2\mu^2 \frac{\partial E(\mu)}{\partial \mu}} + \mu \frac{\partial E(\mu)}{\partial \mu} + E(\mu) \tag{5}$$

where the parameter  $\mu$  is determined from the equation:

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} = \frac{1}{\sqrt{m_1^2 - 2\mu^2 \frac{\partial E(\mu)}{\partial \mu}}} + \frac{1}{\sqrt{m_2^2 - 2\mu^2 \frac{\partial E(\mu)}{\partial \mu}}} \tag{6}$$

and  $E(\mu)$  is the eigenvalue of the nonrelativistic Hamiltonian  $H\Psi(\mathbf{r}) = E(\mu)\Psi(\mathbf{r})$ . Parameters  $\mu_1$  and  $\mu_2$  are mass components of the bound state (constituent mass of particles), which are different from the masses  $m_1, m_2$  of free condition.

Therefore, for the baryons with three quarks with the Hamiltonian  $H\Psi(\mathbf{r}) = E(\mu_i, \mu_j)\Psi(\mathbf{r})$  using above methods we have the bound state mass [21-24]:

$$M_{baryon} = \sqrt{m_1^2 - 2\mu_i^2 \frac{\partial E(\mu_i, \mu_j)}{\partial \mu_i}} + \sqrt{m_2^2 - 2\mu_i^2 \frac{\partial E(\mu_i, \mu_j)}{\partial \mu_j}} + \sqrt{m_3^2 - 2\mu_j^2 \frac{\partial E(\mu_i, \mu_j)}{\partial \mu_j}} + \mu_i \frac{\partial E(\mu)}{\partial \mu_i} + \mu_j \frac{\partial E(\mu_i, \mu_j)}{\partial \mu_j} + E(\mu_i, \mu_j) \tag{7}$$

Where

$$\mu_1 = \sqrt{m_1^2 - 2\mu_i^2 \frac{\partial E(\mu)}{\partial \mu_i}}, \mu_2 = \sqrt{m_2^2 - 2\mu_i^2 \frac{\partial E(\mu)}{\partial \mu_j}}, \mu_3 = \sqrt{m_3^2 - 2\mu_j^2 \frac{\partial E(\mu)}{\partial \mu_j}} \tag{8}$$

and

$$\frac{1}{\mu_i} = \frac{1}{\mu_1} + \frac{1}{\mu_2}, \quad \frac{1}{\mu_j} = \frac{1}{\mu_1 + \mu_2} + \frac{1}{\mu_3} \quad (9)$$

#### 4. Radius of hadronic nuclei

In this section, we determine radius of hadronic nuclei based on amount of overlap volume in 3DL model for some of nuclei. Radius of the pion is  $0.672fm$  with mass  $139.57 MeV$  and radius of the proton is  $0.879fm$  with  $938.24MeV$  [25-27]. The quark mass and radius presented in table1. Using equation (5) and (8) we obtained the bound state mass and radius of hadronic nuclei as presented in table 2. The results show that charged radius of mesons and baryons increase with increasing of the bound states mass. As we suppose colored charge radius for all types of quarks is the same, so we describe some interesting idea about colored charge radius. Increasing of hadron's charge radius could be depended on the mass of quarks and colored charge radius. On the other words, colored force (confinement) could be directly depend on the mass, i.e. the rest mass, the constituent mass and the bound state mass of quarks [28-33].

**Table 1:** Quarks mass and radius.

	$m_q(MeV)$	$r_q(fm)$
$q_u$	2.3	0.00689
$q_d$	4.8	0.004507
$q_s$	95	0.001007
$q_c$	1275	0.0002478
$q_b$	4180	0.00013098
$q_t$	$173.07 \cdot 10^3$	0.00002041

**Table 2:** The bound state mass and radius of hadrons in the ground states.

	$m(MeV)$	$r_q(fm)$	$r_q(fm)$ other authors
$\pi^\pm(u\bar{d}, \bar{u}d)$	139.84	0.6721	0.672[34]
$\kappa^\pm(s\bar{u}, u\bar{s})$	494.77	0.532	0.655[33]
$D^+(c\bar{d})$	1869.51	0.496	0.337-0.219[33]
$D_s^+(c\bar{s})$	1968.12	0.521	0.352-0.425[35]
$B^+(u\bar{b})$	5278.893	0.738	0.614-0.704[35]
$B_c^+(c\bar{b})$	6276.42	0.878	0.658[35]
$P(uud)$	938.64	0.879	0.879[36]
$\Lambda_c^+(udc)$	2286.4635	1.043	-
$\Sigma^+(uus)$	1189.26	0.747	0.774-0.901[35]
$\Sigma^-(dds)$	1197.37	0.752	0.781-0.883[35]
$\Sigma_b^+(uub)$	5811.59	1.062	0.821-0.708[35]
$\Xi^-(dss)$	1321.91	0.829	0.707[37]
$\Xi_c^+(usc)$	1321.56	0.914	-
$\Xi_{cc}^+(dcc)$	3518.23	1.216	-
$\Omega_b^-(bss)$	6071.25	1.105	0.624-0.883[35]

#### Conclusion

Considering that the strong interaction effects between quarks of a meson and baryon can be approximately described harmonic nuclei creations. We derived a method, which based on 3DL model that predicate the radius of hadrons. Using it we can obtain a formula for the description of the radii of mesons and baryons. The calculation is very consistent and agrees with the other defined value for the ground state of hadronic nuclei. Strong interactions inside hadrons started from quark-gluon interactions. Quark colored charge creates spherical colored space around the quark matter. The color charged force directly depends on the mass of quarks that becomes stronger with growth quark matter. As we supposed that this radius is the same in all types of quarks so strong interaction must be occurred by overlapping these colored charge space. In 3DL model quark's interactions should be arranged in the three-dimensional coordinate in order to organize spherical-symmetric shape for hadronic nuclei. Each nucleon inside the nuclide bonds with the neighboring nucleons in xy and xy/z directions based on hadronic cores. We presented new parameters: the number of overlaps and the baryon's symmetric coefficient mass (mass correction's coefficient). Thus, we calculated the charged radius of hadrons. The results very well confirm with experimentally and theoretically calculation of other authors.

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