The Caspian Sea Journal
ISSN: 1578-7899

Volume 10, Issue 1, Supplement 4 (2016) 410-414

Predicting Pattern Formation of Mesons

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ABSTRACT graph theory is a branch of mathematics which developed slowly over the different fields of sciences. Graphical models have various applications in basic and engineering sciences which include statistical physics, solid state physics, bioinformatics, telecommunication and etc. Finding its origins in modern physics, this field would finally be applied strongly to particle, quantum physics and quantum computing. Usage of graph theory in particle physics needs complex computations in order to evaluation of various functions, so there are some powerful methods including field theory method, characteristic of interactions between elementary particles and etc. Quarkonium or mesonic bounding states by graphical models could be described and developed hadronic physics, which is possible by generalization of interactions probability theory to bound states characteristics. Indeed, graph theory has the advantage that it contains easily formulated issues that can be stated in the quantum chromodynamics theories. Finding possible interactions to any two of six quark flavours by using graph theory it is very important. Therefore, the main goal of this paper is preparing a primary generalization of mesonic graph, as a type of graphical models, to bounding case and applying in particle interactions in high energy physics.
KEYWORDS: Mesons, quarkonium, graph theory, adjacency and incidence matrix matrices.
ABBREVIATION: Quantum chromodynamics, Quantum field theory

Introduction

Graph Theory was born in 1736 during the path for solving the solution of a problem relating to the theory of position by Euler. The word graph appeared for the first time in the context of natural sciences in 1878, when the English mathematician James J. Sylvester wrote a paper entitled "Chemistry and Algebra" which was published in Nature. The use of graph theory and group theory in physics, pioneered by many and physical graph theorists is today well established; it has become even more popular after the recent discovery of grapheme and new elementary particles bounding states. There are few areas of physics today in which graphs and group are not involved directly or indirectly [1, 2]. The first thing that needs to be clarified is that the term graph is used indistinctly in the mathematical concept, in general referred to small, artificial formations of nodes and edges. Therefore, it is clear that we will refer to the system of individuals and their interactions as systems. In the tight-binding approach for studying bounding states of quarks, the interaction between quarks generation is not neglect able. Study mesons creation by using vertices and edges in quark diagram, which is known as the graph theory method in mathematics, can be very useful and simple. Here, we concentrate our discussion on alternant conjugated quark and antiquark states in which consider a mesonic (a hadronic system consists of light quark and its own antiquark) and a quarkonium states (a hadronic system consists of heavy quark and its own antiquark) [3-5]. Quantum chromodynamics (QCD) lies at the root of modern physics. One of the most pressing open questions is how to determine exactly six quarks flavour bounding states. There are several attempts for this, among which quantum field theory (QFT) and Feynman graphs are the best known. In each of these attempts one of the key problems is long mathematical calculations and accordance of laboratory experimental results with theoretical results. What is certainly more fundamental and simpler than QFT? We can find good communication between graph theory and particle physics as it finds long time ago between group theory and particle physics. The hadronic-world phenomena in particle physics is best known as the six flavours of quarks, i.e. at least two quarks are bounded to each other by strong interactions that named after mesons or mesonic system like Pion, Psion, Upsilon etc. Quarkonium bounding states systems in graph theory are represented by quarks (nodes/vertices) and quark interactions (links/ edges) between the two flavours of quarks. If there are two nodes without a direct link, we could possibly take other routes based on possible strong interactions that related to quark mass and quark generations. The most possible number of interactions need to be in own generation for light quarks and could be on own and lower generation for heavy quarks. Quarks have three generation that includes in "Tab.I".

Table 1. Quarks generation by their mass

	First generation	Second generation	Third generation
Quark flavor	*u, d*	*s, c*	*b, t*
Quark mass	2.3MeV~4.8MeV	95MeV~1.2GeV	4.2GeV~173GeV

First generation is light and third generation is heavy. As we know one of the benefits of graph theory is to give a unified formalism for different looking issues in basic sciences like particle physics. This can lead us to the birth of the eigen-diagrams in hadronic physics, the so-called quarks graphical diagram. Definition of graph theory with eigen-diagrams of quarks in strong interactions between quarks generation applicable to problems of bounding states creation which are treated in this article. In this quark's diagram, line structures have a large role to creation possible mesonic states. The role of quark's graph diagrams is only in visual-imaging strong interactions, so this cooperation is what makes graph theory and graph diagram so interesting in particle physics. This article show us a brief introduction to bound states interactions based on graph theory and graph diagrams. Using the opportunity of visual –imaging method of quark-quark interactions given by matrices presentations of graphs, we can describe possibility of bounding states with six flavours of quarks and its antiquarks systems.

Definitions of graph in particle physics

We need to clarify the terms graphs and quarks diagram based on mathematical concept, in general referred to possible formations of (nodes/vertices) and (links/ edges). The term quarks diagram is presented for the graphs representing mesonic objects in which the nodes represent entities of the quark and the links represent the possible relationships among them. Therefore, based on hadronic interactions we have to determine exact links between quarks generation that could be create mesonic bound state systems [6-9].

It is impossible to create all possible mesonic systems that we can link in quark's graph diagram, because of the quark's short lifetime. Example we have not mesons consist of u-quark and t-antiquark and T- mesons are not expected to be found in nature. Now, let us consider a finite set $V = \{v_1, v_2, ..., v_n\}$ of unspecified elements and let $V \otimes V$ be the set of all ordered pairs (v_i, v_j) of the elements of V. A relation on the set V is any subset $E \subseteq v \otimes v$. The relation E is symmetric if $(v_i, v_j) \in E$ implies $(v_j, v_i) \in E$. The relation E is anti-reflexive if $(v_i, v_j) \in E$ implies $(v_i \neq v_j)$. Now we can define a simple graph as the pair $G = (V, E)$, where V is a finite set of nodes, vertices or points and E is a symmetric and anti-reflexive relation on V, whose elements are known as the edges or links of the graph.

In a directed graph the relation E is non-symmetric. Formally, in hadronic physics quarks generation based on graph theory and diagram is a pair of sets $G = (V, E)$, where V is the set of vertices and E is the set of edges, formed by pairs of vertices. E is a multiset, in other words, its elements can occur more than once so that every element has a multiplicity. There is unique way of drawing mesonic graphs that we will present it in the next paragraphs.

So, we label the quark vertices with letters (u, d, s, c, b, t) and (u*, d*, s*, c*, b*, t*) as it shows in "Fig. 1". Using graph theory we have the following principles in quark's graph diagram. The two vertices q and q* are end vertices of the edge (q, q*) i.e. mesonic state.

An edge of the form (q, q) is a null loop, i.e. E is empty. A graph can linked two nodes just if they are in the same generation or linked with nodes from the lower generations. Edges are adjacent if they share a common end vertex. Two vertices q and q* are adjacent if they are connected by an edge i.e. (q, q*) is an edge. The degree of the vertex q, written as d(q), is the number of edges with q as an end vertex [9, 11].

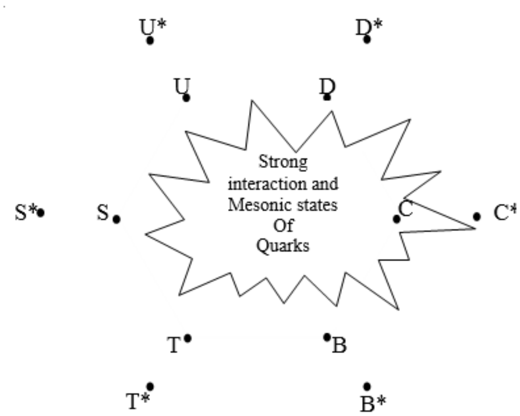

Fig.1: Quark's and antiquark's graph diagram. Black point is the graph nodes.

Therefore, there is no mesonic state with those nodes if a pendant an isolated vertex (or whose degree is zero). The quarks diagram can be presented as a bipartite graph that presented in "Fig. 2".

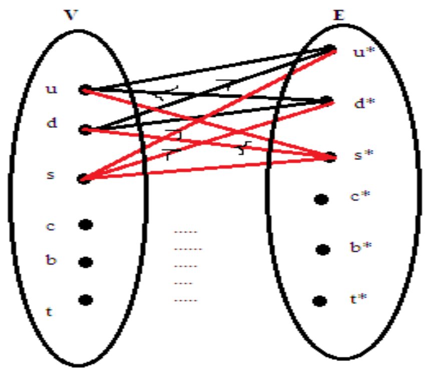

Fig.2: Bipartite graph.

In bipartite graph vertex set can be partitioned into two sets V and E. Each edge has one end in V and one end in E. Therefore, based on graph theory we can describe quarks diagram by bipartite graph and also by circle two demission graphs “Fig.3”.

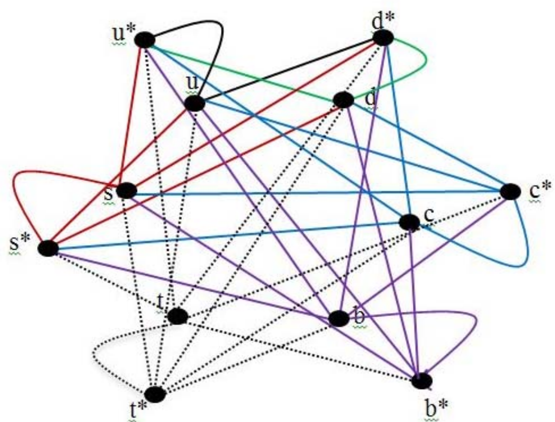

Fig.3: Quark’s graph diagram in two dimensions. First dimension is quarks and second dimension is antiquarks.

Adjacency and incidence matrices of mesons

In the last decade large progress has been made in the field of light and heavy mesonic physics. Several experiments, contributed to increase the data on mesonic spectra. A few conventional mesonic states were discovered and some mesonic states were found outside the quark model. QCD is the part of the standard model of particle physics that deals with the strong interaction. It is a Yang-Mills theory based on the SU(3) group theory with six Dirac fields (quarks) of different masses. Now we want to introduce mesonic states creation by using graph theory that presented in the above paragraph. Therefore, we have to introduce some of graph theory terminologies in particle physics like incidence and adjacency matrix for quark’s graph diagram. In particle physics as we can see in “Fig. 2” and “Fig.3” some of connection /bound states of mesonic system could not be created because of some strong interaction principles in QCD. These uncreated states can be presented in graph theory by using definition of direct and indirect graph and etc. in particle physics and strong interactions. Therefore, based on graph theory, we can show that some quark’s graph diagrams has special name depending on their specifications and particularity. One of the key distinctions people make between quark’s graphs is whether they are directed or undirected. When we talk about undirected quarks’ graphs, really, all we are saying is whether the edges in a graph are bidirectional or not. Most, but not all, quark’s graphs have only one kind of edge, i.e. dependence on their generations, quarks bounding states could be just on own generation or could be on own generation and lower generation. This low gets us to the meaning of directed or undirected graphs. In Figure 3, direction of quark’s bounding states denoted by colored lines and we can determine in-degree and out-degree of vertices. So, based on graph theory now, we have to describe direct and indirect connection or on the other words we have to determine which of mesonic states could be created at high energy physics. After long and many discussions, based on the strong interaction principles one can finally present creation of mesonic states using graph theory and propounding it by matrices**.** The mesonic family can represent in the graph through matrix. However, there are many methods to represent a graph in. Each method has its own advantages. In this article, author will only discuss two of the methods that are quite well known, namely adjacency and incident quark's graph matrix. The quark's graph family argues that one of the best ways to represent them into a matrix is by counting the number of edge between two adjacent vertices. Two vertices is said to be adjacent or neighbor if it support at least one common edge. Quarks' graph in "Fig. 3" has twelve vertices in two dimensions i.e., six vertices in each dimensions. Thus, we make adjacency matrix of size 12 (6*2) by 12(6*2). Then we put the name of vertices on the side of the matrix. To fill the adjacency quark's matrix, we look at the name of the vertex (quarks) in row and column (antiquarks):

$$AG_{ij} = \begin{bmatrix} & u & d & s & c & b & t & u^* & d^* & s^* & c^* & b^* & t^* \\ u & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ d & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ s & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ c & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ b & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ t & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} \\ u^* & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ d^* & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ s^* & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c^* & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b^* & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t^* & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

If those vertices are connected by an edge or more, we count number of edges and put this number as matrix element. Based on Fig.3 we start our description about quarks interactions and creation of mesons. Quarks and antiquarks have one common edge, we say that Quarks and antiquarks are mesonic states i.e. they are adjacent. Therefore, we can say that if we have vertices (six quark flavors and six antiquark flavour): $V = \{u, u^*, ..., t, t^*\}$ and edges: $E = \{e_k = q_i q_j^*\}$, adjacency matrix: $AG_{ij} = [Q_{ij}]$ where Q_{ij} is the number of edges joining q_i and q_j. In the quark's adjacency matrix elements $\tilde{1}$ describe mesonic states that could not be created until this moment because of theirs quark's short lifetime [11-14] maybe they will be detected in the near future.

The connection between edge and vertices in graph diagram is the best ways to represent interactions matrix between particles. We can make matrix that related vertices to edges i.e. quarks-antiquarks bound states (edges). A vertex is said to be incident to an edge if the edge is connected to the vertex. If we have vertices (six quark flavors and six antiquark flavour): $V = \{u, u^*, ..., t, t^*\}$ and edges: $E = \{e_k = q_i q_j^*\}$, incidence matrix: $M_{ij} = [q_i q^*_j]$ where $q_i q_j^*$ is the mesonic bound states or created mesonic system. Let us start again our work with quark's graph diagram. Quark's graph "Fig. 3" has 12 vertices and interactions edges. Thus, we make incident matrix of size 12 (6*2) by 12(6*2). The rows represent the quark and antiquark vertices; the columns are mesonic state i.e. the strong interaction's edges. Then we put the name of vertices and edges on the side of the matrix [12-14] (first row and first column is not matrix element they just show quarks bound states and quark flavor):

$$M_{ij} = \begin{bmatrix} & u & d & s & c & b & t & u^* & d^* & s^* & c^* & b^* & t^* \\ u \to qq^* & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ d \to qq^* & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ s \to qq^* & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c \to qq^* & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b \to qq^* & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t \to qq^* & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ u^* \to q^*q & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ d^* \to q^*q & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ s^* \to q^*q & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ c^* \to q^*q & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ b^* \to q^*q & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ t^* \to q^*q & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} \end{bmatrix}$$

In matrix definition adjacency matrix there is an edge (i, j) and an edge (j, i), so $A_{G_{ij}}$ is the symmetrical matrix so as we can see from adjacency matrix of quark's graph diagram is unsymmetrical and also it is directed graph in relation with quark's mass generation as it is explained above. Based on quark's adjacency matrix and incidence matrix we can determine some of mesonic states that presented in below as matrix "A". (These mesons could be created in experimental laboratory of High Energy physics in CERN or KEK based on different quantum number like spin, total angular momentum and etc.) (first row and first column is not matrix element they just show quarks bound states and quark flavor):

$$A=\begin{bmatrix} & u & d & s & c & b & t & u^* & d^* & s^* & c^* & b^* & t^* \\ u\to qq^* & \pi^0 & \pi^+ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ d\to qq^* & \pi^- & \pi^0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ s\to qq^* & \kappa^- & \overline{\kappa}^0 & \phi^0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c\to qq^* & \overline{D}^0 & D^+ & D_s^+ & J/\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b\to qq^* & B^- & \overline{B}^0 & \overline{B}_s^{*0} & B_c^{*-} & Y & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t\to qq^* & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ u^*\to q^*q & \pi^0 & \pi^- & 0 & 0 & 0 & 0 & \pi^0 & \pi^- & 0 & 0 & 0 & 0 \\ d^*\to q^*q & \pi^+ & \pi^0 & 0 & 0 & 0 & 0 & \pi^+ & \pi^0 & 0 & 0 & 0 & 0 \\ s^*\to q^*q & \kappa^+ & \kappa^0 & \phi^0 & 0 & 0 & 0 & \kappa^+ & \kappa^0 & \phi^0 & 0 & 0 & 0 \\ c^*\to q^*q & D^0 & D^- & D_s^- & J/\psi & 0 & 0 & D^0 & D^- & D_s^- & J/\psi & 0 & 0 \\ b^*\to q^*q & B^+ & B^0 & B_s^{*0} & B_c^{*+} & Y & 0 & B^+ & B^0 & B_s^{*0} & B_c^{*+} & Y & 0 \\ t^*\to q^*q & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} \end{bmatrix}$$

Conclusion

In modern physics it is great efforts to understand how bound state arise and which bounding system could be created in the formalism of quantum field theory and quantum chromodynamics. It is very important to work out effective methods to describe, calculate and determine all characteristics of these bound states, especially lifetime and their masses. The analysis of a bound state is simplest when we can be exactly known about possible systems when quarks and antiquarks travel at speeds considerably less than "c". New importance to the bound state physics was given by the development of QCD, the modern theory of strong interactions by using basic sciences subjects like group and graph theories. At the present time the technical achievements of experimental on mesonic systems studies make it possible to obtain the exotic mesons with heavy and light quarks, so the determination of possible relativistic systems becomes necessary. In this article we have considered this problem, according to the (nodes/vertices) and (links/ edges) behaviour of the quarks interactions in the quantum chromodynamics field. Taking account of quarks and interactions between them, determined possible mesonic systems i.e. the strong force could be described by SU(3) symmetry in group theory and bounding states by nodes-links in graph theory. We have investigated the relationship between mesons creations and graphs and presented how the principles of strong interactions between quarks could be predicted by quark's graph diagram. In quark's graph diagram we have shown that how generations of quark can be affected on creation of mesonic system. Using the adjacency quark's matrix and incidence matrix we have described possible and impossible mesonic systems.

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