The Caspian Sea Journal
ISSN: 1578-7899
Volume 10, Issue 1, Supplement 4 (2016) 244-246

# Defining the Point Change of the Distribution Time for the Clients' Reference to Bank

Narges Abbasi,
Department of Statistics, Payame Noor University, I. R. Iran
*Corresponding Author: n_abbasi@pnu.ac.ir
Lela Nazari Sabooki,
Department of Statistics, Payame Noor University, I. R. Iran
Mohsen Emami,
Department of Computer Engineering and IT, Payame Noor University, I. R. Iran
Zahra Al'arab Moosa
Department of Statistics and IT, Shiraz Municipality, I. R. Iran

**ABSTRACT**– Queuing systems are studied specially to prevent the server's time waste when he (she) has nothing to do and shorten the queue time for the client. The presence and definition of the change point are very sensible in the enjoyment increase. In this article the data gathered from the clients' entrance into Bank Tejarat were examined on a defined day in order to define the distribution change on a due time for the Poisson distribution random variables. The change points for the data entered into two independent counters were defined by Gibbs sampling method and MATLAB software in the bank.
**EYWORDS**: Poisson distribution, Gibbs sampling, Bayes theory, maximum likelihood estimation.

## Introduction

Gibbs sampling was stated for the first time in the Gibbs distribution analysis on Lattices by Gemen and Gemen in 1984 and its application was extended by Gelfend and Smith (1990) and Gelfend et al. (1990). Based on Gibbs method this article shows how to define the change point in the Poisson distribution family for the gained data. So with change point Poisson model is the first part of the inputs from Poisson distribution with $\theta$ average until k time and is the distribution number of the Poisson distribution inputs with mean rate from $\lambda$ after $k^{th}$ time.

$X_i \sim poisson(\theta) \qquad i = 1, \dots, k$

$X_i \sim poisson(\lambda) \qquad i = k+1, \dots, n$

Previous distribution is taken into consideration for the Poisson distributions parameters as follows:

$\theta \sim Gamma(a_1, b_1) \quad \lambda \sim Gamma(a_2, b_2) \quad b_1 \sim Gamma(c_1, d_1) \quad b_2 \sim Gamma(c_2, d_2)$

In software calculations above constants are defined as follows:

$a_1 = 0.5;\ a_2 = 0.5;\ c_1 = 0;\ c_2 = 0;\ d_1 = 1;\ d_2 = 1$

Also the variable k is considered with discrete monotone distribution on the set: $\{1, \dots, 67\}$ . The random variables $\theta, \lambda, k$ are considered independent from each other. This model leads to following conditioned distribution:

$$\theta | X, \lambda, b_1, b_2, k \sim Gamma\left(a_1 + \sum_{i=1}^{k} X_i\, , k + b_1\right)$$

$$\lambda | X, \theta, b_1, b_2, k \sim Gamma\left(a_2 + \sum_{i=1}^{k} X_i\, , n - k + b_2\right)$$

$$b_1 | X, \theta, \lambda, b_2, k \sim Gamma(a_2 + c_1, \theta + d_1)$$

$$b_1 | X, \theta, \lambda, b_1, k \sim Gamma(a_2 + c_2, \theta + d_2)$$

$$f(k | X, \theta, \lambda, b_1, b_2) = \frac{L(X; k, \theta, \lambda)}{\sum_{j=1}^{n} L(X; j, \theta, \lambda)}$$

Where the likelihood function is as follows:

$$L(X; k, \theta, \lambda) = \exp\{k(\lambda - \theta)\} \left(\frac{\theta}{\lambda}\right)^{\sum_{i=1}^{n} X_i}$$

You can read the theoretical discussion and calculations in detail in [1].

## Materials and Methods

### *Sample preparation*

The data include the entrances to a Bank Tejarat branch on Tuesday. The data from the clients were taken into consideration when the latter referred to the counters number 6 and 17 of the bank; these two counters had different services for the clients. It is supposed that the clients' entrances follow Poisson distribution in the branch. Figures 1 and 2 indicate the entrances every five minutes.

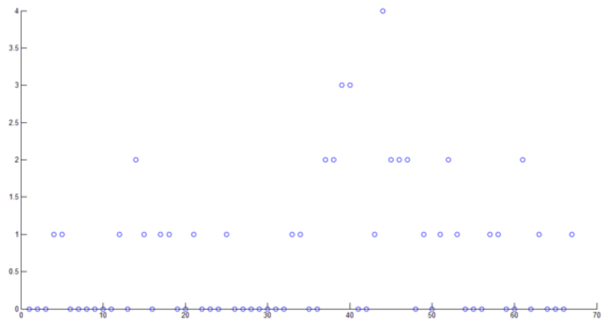

**Figure 1**: The times of the clients' entrances (The time unit is every five minutes) to the counter 6

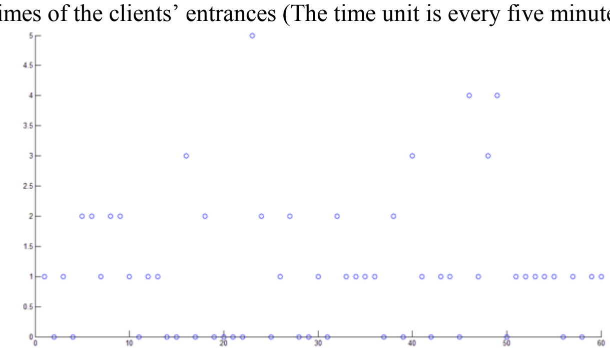

**Figure 2**: The times of the clients' entrances (The time unit is every five minutes) to the counter 17

The descriptive statistics related to the clients to both counters are shown in Table 3; it seems the clients to counter 6 stay more than ones to counter 6 in the bank; both employees' operations are shown in the Table 4. The descriptive findings in the Table 4 indicate the service time in the counter 6 is more than mean time in the 17.

**Table 1**: Descriptive statistics related to the clients' entrances to the counters 17 and 6

| | | The number of clients in a certain day | $mean$ | $max$ | $min$ |
|---|---|---|---|---|---|
| **Counters 17** | **The waiting time to service** | **60** | **0:07** | **0:21** | **0:00** |
| | **Residence time in the bank** | | **0:11** | **0:32** | **0:01** |
| **Counters 6** | **The waiting time to service** | **67** | **0:07** | **0:20** | **0:00** |
| | **Residence time in the bank** | | **0:14** | **0:37** | **0:02** |

**Table 2**: Operations in both counters in the bank

| | **MIN** | Mean | **MAX** |
|---|---|---|---|
| **Counters 6** | 0:02 | 0:14 | 0:37 |
| **Counters 17** | 0:01 | 0:11 | 0:32 |

### *Analytic determinations*

By virtue of Gibbs sampling method we find the change point; now MATLAB program is ready for use. 1,100 repetitions will be for 67 and 60 observations in counters 6 and 17, respectively. The change point dispersion distribution for both counters is shown in the figures 5 and 6. Based on maximum likelihood estimator selection the change point is defined.

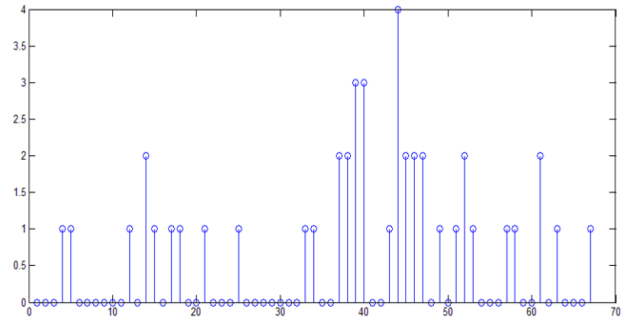

**Figure 3**: Change point definition for the clients of counter 6

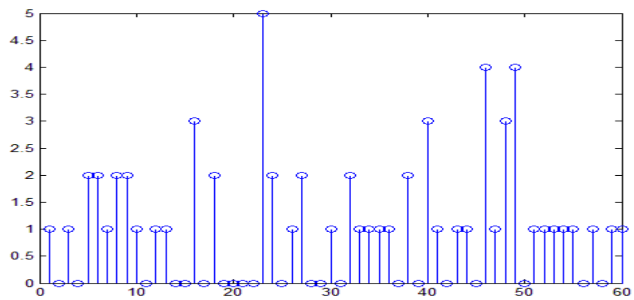

**Figure 4**: Change point definition for the clients of counter 17

Considering there was a change in the scale in the data analysis (One unit every five minutes) so the findings should be converted to primary measurement unit; so by virtue of the findings in Table 7 the clients' change points are $k_6 = 44$ and $k_{17} = 23$ in counters 6 and 17, respectively; these points are 11:10 and 9:25 for the clients to the counters 6 and 17, respectively.

**Table 3**: Change point for the clients to both counters

| Mode k | $\boldsymbol{mean\ of\ \theta}$ | $\boldsymbol{mean\ of\ \lambda}$ | The exact time of Change | |
|---|---|---|---|---|
| 44 | 1:45(0.3485) | 5:11(1.0356) | 11:10 | Cash No. 6 |
| 23 | 5:05(1.0161) | 4:13(0.8418) | 09:25 | Cash No. 17 |

**Results and Discussion**

Overall the findings indicate the clients to counter 6 enter the bank until 11:10 o'clock with shorter mean time than ones to counter 17 while the entrance intervals decrease for the clients to counter 17 after 9:25 o'clock. The entrances to the bank indicate the counter 6 is relatively busy before 11:10 o'clock and then the clients' rate increases because of the clients' entrance rate increase so the queue is shorter; this examination indicates the counter 17 receives more clients after 9:25 o'clock with relatively less rate change, but this situation is not as busy as counter 6. It is recommended the bank management to employ additional employee to counter 6 in early hours to decrease the queue in front of the counter and do the same for the counter 17 after 9:25 o'clock.

**References**

1. Gelfand AE, Smith AFM (1990). "Sampling-based approaches to calculating marginal densities," Journal of the American Statistical Association, 85: pp. 398-409.
2. Geman S, Geman D (1984). "Stochastic relaxation, Gibbs distributions and the Bayesian restoration of images," IEEE Transactions PAMI, 6: pp. 721-741.
3. Martinez W L, Martinez AR (2002) Computational Statistics Handbook with MATLAB, Chapman & Hall/CRC.